Density dependence of the MIT bag parameters from the field theory of hadrons

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Abstract

A self-consistent description of the MIT bag parameters as functions of the nuclear matter density is presented. The subnuclear degrees of freedom are treated in the Quark-Meson Coupling Model, considering the equilibrium conditions for the bag in the nuclear medium. The hadronic interaction is described in the framework of the quantum field theory of hadrons through several models. We have obtained the behavior of the bag radius and the bag parameters B and z_0, taking their derivatives with respect to the mean field value of the scalar meson as free parameters. A discussion on the variation range of these derivatives is given.

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It is well known that Quantum Chromodynamics (QCD) is the most accepted candidate for a theory of strong interactions. Although QCD has been successful in describing the high energy domain, its application seems difficult in the energy range associated with nuclear phenomena. Attempts to sort this difficulty have been carried out by using effective models, among them the so-called bag models are widely used [1, 2]. They provide an acceptable description of the free hadronic properties. The energy-momentum conservation of an isolated bag is imposed through the so-called non-linear boundary condition [1, 2].

The Quark-Meson Coupling Model (QMC) has been developed since the early work of Guichon [3]. In this model the nucleons are described as non-overlapping MIT bags confining the quarks inside them. The quarks interact through the exchange of scalar (σ) and vector (ω_μ) mesons. Several refinements and applications of this model have been done [4]–[7], for instance the QMC model has been used to describe nuclear and neutronic matter [4, 5], as well as finite nuclei [5, 6].

On the other hand, the nuclear phenomena has been treated in the context of quantum field theory by using structureless hadronic fields as the effective degrees of freedom. This subject reached interest since the pionnering work of Walecka [8]. Different aspects of the phenomenology of nuclear matter and finite nuclei have been studied in the framework of the so-called Quantum Hadrodynamics (QHD) with successful results [9]. The original Walecka model has been extended with additional polynomic potentials [10] and alternative non-polynomic interactions in the scalar channel [11]-[14].

If the validity of the equation of state , effective mass, etc., derived from an effective hadronic lagrangian is assumed, one can ask about the hadronic substructure consistent with those properties. We are interested in the implications that the QHD description have for a picture which deals with subnuclear degrees of freedom. A clear relationship between the QMC model and the Walecka model has been stablished by Saito and Thomas [4] . In the present work we have studied the reliability of a coherent description of in-medium nucleon properties in terms of QHD and QMC.

We briefly recall some basic features of the QMC model [3, 4]. If isospin symmetry breaking is neglected the meson fields $\sigma(x)$ and $\omega_\mu(x)$ are sufficient to describe the problem. Inside the bag the equation of motion for quarks of mass m_q is given by

$$(i\not{\partial} - m_q)\Psi_q(x) = [-g^q_\sigma \sigma(x) + g^q_\omega \not{\omega}(x)]\Psi_q(x), \tag{1}$$

where g^q_σ and g^q_ω are the quark-meson coupling constants associated with the σ and ω_μ fields, respectively. In the Mean Field Approximation (MFA) the meson fields are replaced

by their mean values, which become constants in infinite nuclear matter, i.e. $\sigma = \bar{\sigma}$ and $\omega_\mu = \bar{\omega}\delta_{\mu 0}$.

The normalized quark wave function for a spherical bag of radius R is

$$\Psi_q(\vec{r}, t) = \mathcal{N} e^{-i\epsilon_q t/R} \times \begin{pmatrix} j_0(yr/R) \\ i\beta_q \vec{\sigma}\cdot\hat{\mathrm{r}} j_1(yr/R) \end{pmatrix} \frac{\chi_q}{\sqrt{4\pi}}, \tag{2}$$

where $r = |\vec{r}|$, χ_q is the quark spinor and the normalization constant $\mathcal{N}$ is given in [3].

We have introduced the effective quark mass $m_q^* = m_q - g_\sigma^q \bar{\sigma}$ and the effective energy eigenvalue $\epsilon_q = \Omega/R + g_\omega^q \bar{\omega}$, where $\Omega = \sqrt{y^2 + (R m_q^*)^2}$. The y variable is fixed by the boundary condition at the bag surface $j_0(y) = \beta_q j_1(y)$ as in reference [1] and $\beta_q = \sqrt{(\Omega - R m_q^*)/(\Omega + R m_q^*)}$.

The bag energy is given by

$$E_b = \frac{3\Omega - z_0}{R} + \frac{4}{3}\pi B R^3 \tag{3}$$

where B is the energy per unit of volume and z_0 takes into account the zero point energy of the bag. The nucleon mass is defined by including the correction due to the spurious center of mass motion [7]

$$M_b^* = \sqrt{E_b^2 - 3(y/R)^2}. \tag{4}$$

It is usual to determine the bag parameters at zero baryon density to reproduce the experimental nucleon mass $M_b = 939$ MeV. Simultaneously it is required the equilibrium condition for the bag $dM_b(\bar{\sigma})/dR = 0$. In [5] the bag parameters B and z_0 are constants, although it was found a relative change of 1% in the radius at the saturation density ($\rho = 0.15$ fm^{-3}) as compared with its value at zero baryon density.

In the normal QMC model the bags interact by the same mechanism that couples mesons to quarks inside a bag. In the present work we have proposed a description of the hadronic interaction by a set of different effective models commonly used in QHD. In all the cases considered we have used a linear coupling between the nucleon field $\Psi(x)$ and the vector-meson field, with strength g_ω. However the models differ in the interaction term $\bar{\Psi}(x) V_{N\sigma} \Psi(x)$, between the nucleon and the scalar meson field. In Table I we show the explicit form of $V_{N\sigma}$ for these four models. The coupling constants g_σ and g_ω have been fixed to reproduce the binding energy per nucleon, $E_b = 16$ MeV, and the nuclear saturation density, $\rho = 0.15$ fm^{-3}, in the MFA. The isothermal compressibility $\kappa = 9\rho\,(\partial P_h/\partial \rho_B)_T$ is a usefull quantity in order to analyze the fitness of hadronic models, since its value at the nuclear saturation density has been well determined to range between

$100 - 300$ MeV [15]. The isothermal compressibility evaluated in the MFA is also shown in Table I [14].

The Euler-Lagrange equations for nucleons and mesons in the MFA are given by

$$(i\partial\!\!\!/ - M_N)\Psi(x) = (g_\omega \bar{\omega}\gamma_0 - V_{N\sigma})\Psi(x), \tag{5}$$

$$m_\sigma^2 \bar{\sigma} = \frac{dV_{N\sigma}}{d\sigma}(\sigma = \bar{\sigma}) \;\; \rho_s, \tag{6}$$

$$m_\omega^2 \bar{\omega} = g_\omega \rho, \tag{7}$$

where $\rho_s =< \bar{\Psi}(x)\Psi(x) >$, ρ is the baryon density; M_N, m_σ and m_ω are the masses of the free nucleon, the scalar and vector mesons, respectively.

At zero temperature we have

$$\rho = \frac{2k_F^3}{3\pi^2}, \tag{8}$$

and

$$\rho_s = \frac{4}{(2\pi)^3} \int d^3\vec{k}\Theta(k_F - |\vec{k}|) \frac{M_N^*}{\sqrt{{M_N^*}^2 + \vec{k}^2}}, \tag{9}$$

where k_F is the nucleon Fermi momentum. The effective nucleon mass M_N^* and the energy spectrum $\epsilon(k)$ are given by

$$M_N^* = M_N - V_{N\sigma}, \tag{10}$$

and

$$\epsilon(k) = \sqrt{{M_N^*}^2 + k^2} + g_\omega \bar{\omega}. \tag{11}$$

We have used different notations for the nucleon mass entering in QHD (M_N^*), and the nucleon mass generated by the bag model (M_b^*).

Equation (6) is a self-consistent definition for the mean field value $\bar{\sigma}$, indeed from this equation we see that the derivative $dM_N^*/d\bar{\sigma}$ determines the dynamics of the scalar field. The QMC and QHD descriptions produce coherent results if the following equation is fulfilled

$$M_N^*(\sigma) = M_b^*(\sigma), \tag{12}$$

together with $g_\sigma = 3g_\sigma^q$, $g_\omega = 3g_\omega^q$, [4].

The stability of the bag in the nuclear medium with respect to volume changes is imposed by

$$P_b(\sigma) = P_h(\sigma), \tag{13}$$

where $P_b(\sigma)$ is the internal pressure generated by the quark dynamics and $P_h(\sigma)$ is the external hadronic pressure.

The bag pressure can be written as

$$P_b = -\frac{E_b}{4\pi R^2 M_b^*}\left(-\frac{E_b}{R} + \frac{16}{3}\pi R^2 B + \frac{3m_q^{*2}}{\Omega}\right) - \frac{3y^2}{4\pi R^5 M_b^*}, \qquad (14)$$

and the pressure for uniform nuclear matter is given by $P_h = -\frac{1}{3}T^{ii}$, where T^{ii} is the trace over the spatial components of the energy-momentum tensor

$$P_h = \frac{2M_N^{*3}}{\pi^2}\left[\frac{k_F}{M_N^*}\left(\frac{5\epsilon_F}{24} - \frac{\epsilon_F^3}{12M_N^{*2}}\right) - \frac{M_N^*}{8} ln\left(\frac{\epsilon_F + k_F}{M_N^*}\right)\right]. \qquad (15)$$

The equation (13) is a statistical equilibrium condition on the bag surface which ensures a direct relation between nuclear matter bulk properties and the stability of the confining volume.

The equations (12) and (13) can be used to obtain the bag parameters as functions of the density. However, they are not enough to determine all the functions involved. Henceforth we have included the additional equations obtained from the derivatives of the boundary condition and of the equations (12) and (13), taking y, m_q^*, R, z_0 and B as independent functions. In fact we have considered $\lambda = dB/d\bar{\sigma}$ and $\mu = dz_0/d\bar{\sigma}$ as constants. This procedure is equivalent to take only the linear contributions in the expansion of the functional relations (12) and (13).

After the hadronic coupling constants have been adjusted to reproduce the saturation conditions for nuclear matter, equation (6) can be used to obtain the solution $\bar{\sigma}$ for each baryon density. Equations (3), (4), (10) and (12) can be used together with eqs. (13), (14) and (15) to determine B and z_0 when the value of the in-medium bag radius is provided. The last quantity is obtained by solving self-consistently the additional equations for fixed values of the parameters λ and μ. To search for appropriate values of λ and μ we have evaluated R, B and z_0 at zero baryon density as a function of (λ, μ) using the model 1. A drastic change in the behavior of the quantities considered is found when μ goes from negative to positive values.

The bag radius R at the saturation density has been studied as a function of λ. For the hadronic models considered here and taking the bag radius at zero baryon density $R_0 = 0.6$ fm we have found that $R/R_0 > 1$ only for a restricted range of μ. For the following discussion we have taken two sets of values; set I ($\lambda = -5.28$ fm^{-3}, $\mu = -0.50$ fm) and set II ($\lambda = 0, \mu = 1.6$ fm).

The bag radius as a function of the density is shown in fig. 1, the quark mass at zero baryon density has been fixed at $m_q = 10$ MeV. It can be seen that set I gives an asymptotical constant bag radius for every model, whose values do not depend on the

details of the interaction and it is diminished as compared with its vacuum value. The set II provides a model dependent radius at high densities, in this case models 1 and 2 predict a breaking-down of the bag picture. Models 3 and 4 have a stable behavior for all the densities considered here, even when set II is used.

In figs. 2 and 3 we present the density dependence of $B^{1/4}$ and of z_0, respectively. Models 1 and 2 exhibit an opposite behavior for $B^{1/4}$, when set I or set II are used.

For model 1 and set I B takes negative values as the baryon density is sufficiently increased, thus the bag bulk energy must decrease with increasing volume. The increment of the kinetic energy compensates this fact, giving a slowly decreasing total bag energy and a stable bag radius (see fig. 1). On the other hand, B grows drastically at high densities for set II, a small volume increment gives rise to a large increment in the bulk energy. Therefore in order to get a slowly decreasing M_b^*, the bag radius R must decrease at the same rate as $B^{1/3}$ grows. When R approaches to zero a subtle cancellation among the quark kinetic energy, the zero point motion parameter and the center of mass correction takes place. The raising of z_0 (see fig. 3) is not sufficient to reach the dynamical equilibrium and hence the system reduces its volume as far as possible.

The steep behavior of models 1 and 2 as compared with models 3 and 4 is due to the fact that the first mentioned models give a stiff equation of state and a fast decrease for the effective nucleon mass.

Our results can be compared with those obtained by Jin and Jennings [16]. In their work a phenomenological description of B as a function of the density is given in terms of a set of two free parameters with no direct dynamical interpretation, namely g_σ^B, δ and B_0 for model I of [16]. Starting from the usual approach in the QMC model, in ref. [16] the constant B is replaced by a function of the baryon density. Within this framework they found a monotonous density dependence of the bag parameter B. For $\delta = 4$ and $g_\sigma^B = 1$, the ratio B/B_0 decreases 80% at the nuclear saturation density and the corresponding in-medium bag radius increases 60%. In our approach we have obtained an increase of 12% for the bag radius (fig. 1) and a corresponding decrease of 19% in B (fig. 2), when model 4 and the set II of parameters are used.

In this work we have studied the coherence of the QHD and QMC descriptions by using the equilibrium conditions for the bag in nuclear matter. Thus we have stablished a link between these frameworks which is able to explain the nucleon substructure changes taking into account the nuclear matter equation of state, the nucleon effective mass, etc.

We have evaluated the density dependence of the bag parameters and the bag radius.

In our model we use two dynamical quantities, i.e. the derivatives $dB/d\bar{\sigma}$ and $dz_0/d\bar{\sigma}$ as free parameters and we have explored their possible variation range. We have found two different dynamical regimes for these parameters.

The inclusion of thermal effects and the study of the EMC effect in this framework will be reported in future works [17].

References

[1] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. **D10** (1974) 2599.

[2] A. W. Thomas, Adv. Nucl. Phys. **13** (1983) 1.

[3] P. A. M. Guichon, Phys. Lett. **B 200** (1988) 235.

[4] K. Saito and A. W. Thomas, Phys. Lett. **B 327** (1994) 9; K. Saito and A. W. Thomas, Phys. Rev. **C51** (1995) 2757.

[5] K. Saito, K. Tsushima and A. W. Thomas, University of Adelaide Preprint, ADP-96-17/T220 (1996).

[6] P. A. M. Guichon, K. Saito, E. Rodionov and A. W. Thomas, University of Adelaide Preprint, ADP-95-45/T194 (1995), nucl-th/9509034.

[7] M. Betz and R. Goldflam, Phys. Rev. **D 28** (1983) 2848. J. L. Dethier, R. Goldflam, E. M. Henley and L. Wilets, Phys. Rev. **D 27** (1983) 2191.

[8] J. D. Walecka, Ann. of Phys. **83** (1974) 491.

[9] B. D. Serot and J. D. Walecka, Adv. in Nucl. Phys. vol. 16 (1986) 1.

[10] J. Boguta and R. Bodmer, Nucl. Phys. **A 292** (1977) 413.

[11] J. Zimanyi and S. A. Moszkowski, Phys. Rev. **C 42** (1990) 1416.

[12] M. M. Sharma, S. A. Moszkowski and P. Ring, Phys. Rev. **C 44** (1991) 2493.

[13] R. Aguirre, O. Civitarese and A. L. De Paoli, Nucl. Phys. **A 597** (1996) 543.

[14] H. Feldmeier and J. Lindner, Z. Phys. **A 341** (1991) 341.

[15] J. P. Blaizot, D. Gogny and B. Grammaticos, Nucl. Phys. **A 265** (1976) 315. G. E. Brown and E. Osnes, Phys. Rev. **B 159** (1985) 223.

[16] X. Jin and B. K. Jennings, Phys. Lett. **B 374** (1996) 13. X. Jin and B. K. Jennings, nucl-th/9604018 and nucl-th/9606025. X. Jin and B. K. Jennings, TRIUMF Preprint TRI-PP-96-35 (1996).

[17] R. Aguirre and M. Schvellinger, University of La Plata Preprint, La Plata-Th 96/19 (1996), nucl-th/9611028.

Table I.

$Model$	$V_{N\sigma}$	g_σ	g_ω	κ
				[MeV]
1	$g_\sigma \sigma$	11.04	13.74	554
2	$M_N g_\sigma tanh(g_\sigma \sigma / M_N)$	9.15	10.52	410
3	$M_N[1 - exp(g_\sigma \sigma / M_N)]$	8.34	8.19	267
4	$g_\sigma \sigma / (1 + g_\sigma \sigma / M_N)$	7.84	6.67	224

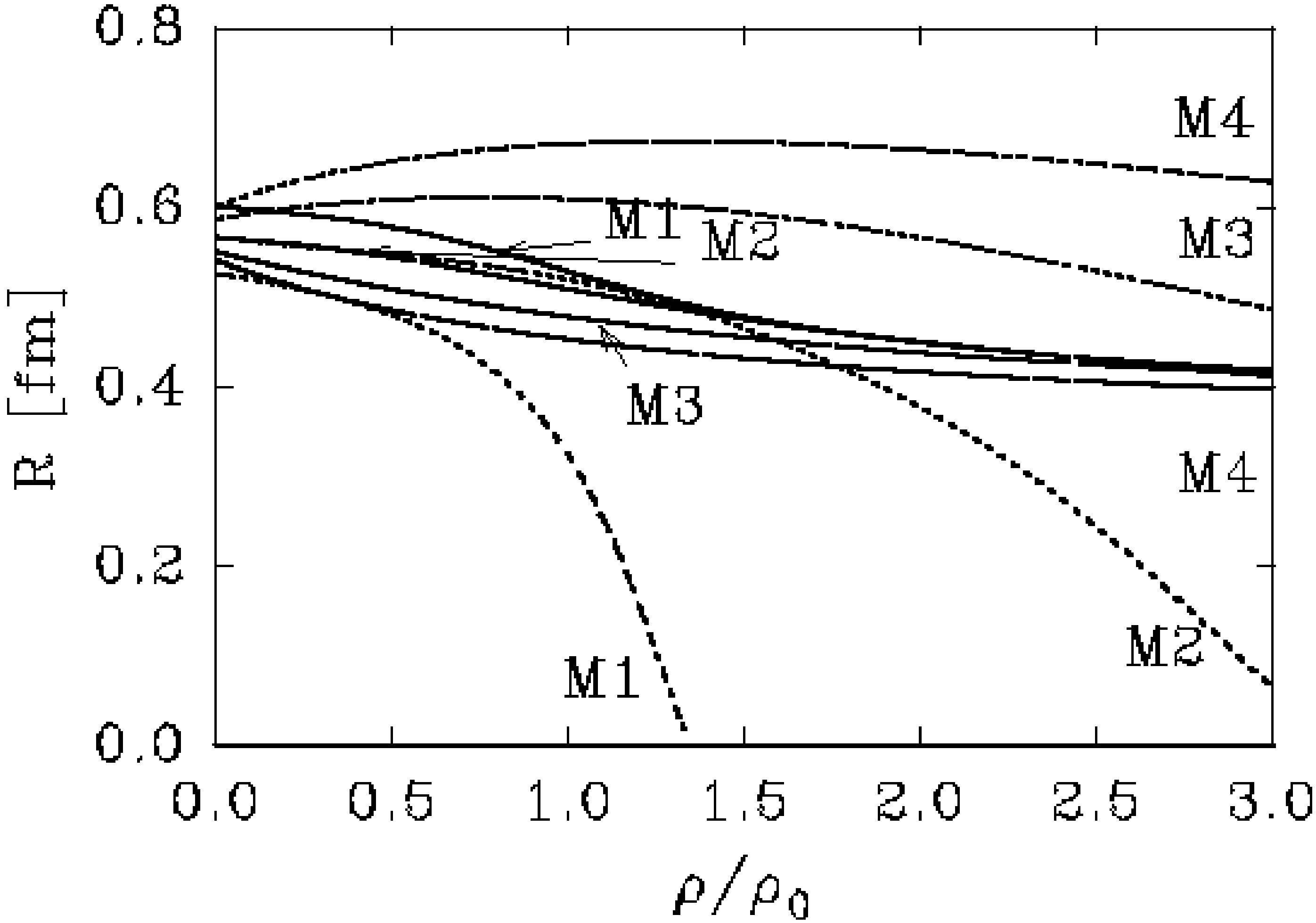
Table I: Nucleon-scalar meson interaction terms, coupling constants and isothermal compressibility κ, for several effective hadronic models used in this work.

Figure Captions

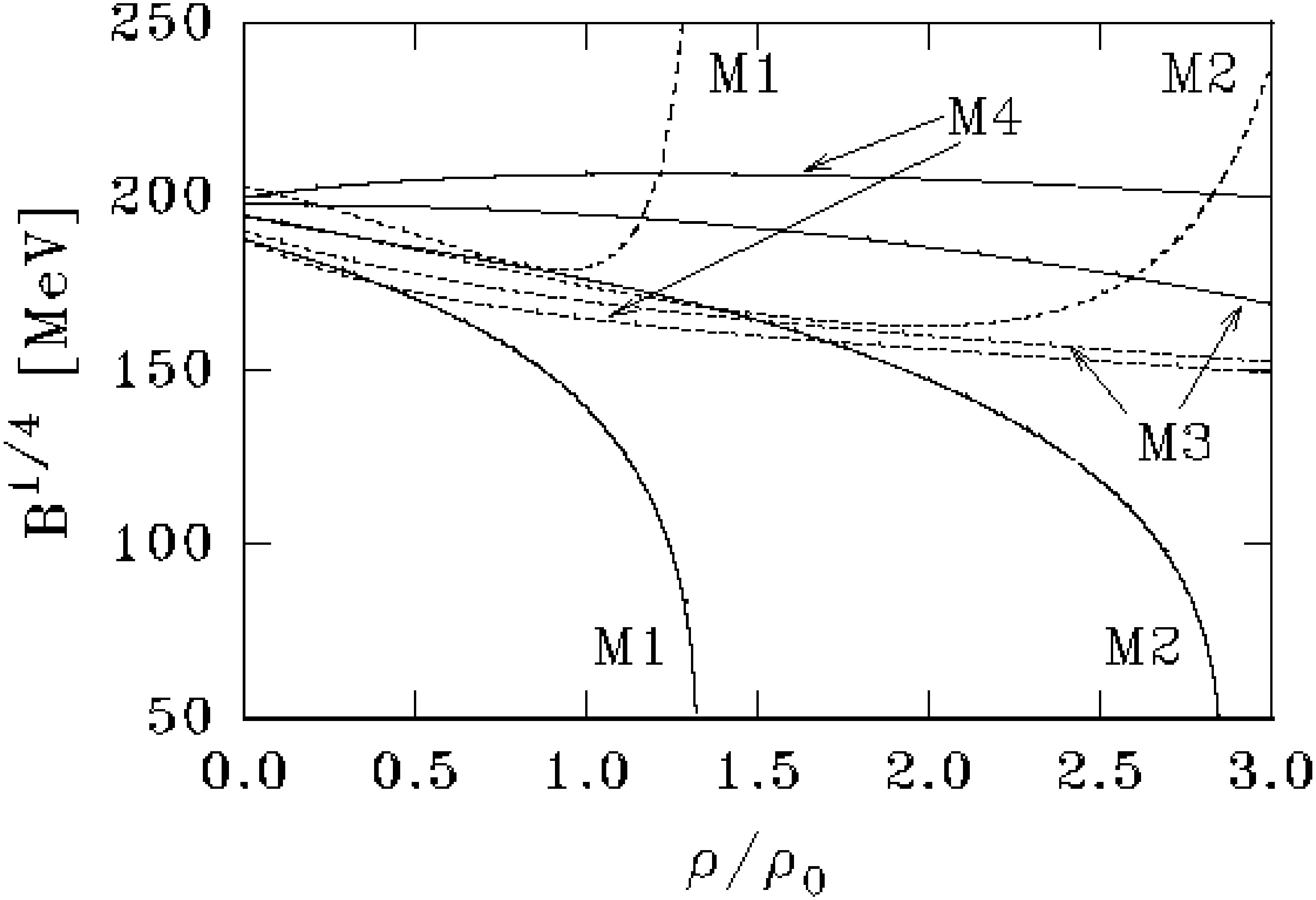
Figure 1: The bag radius R as a function of the relative baryon density ρ/ρ_0, where ρ_0 is the nuclear matter density at saturation, for hadronic models indicated as M 1 (Walecka model), M 2, M 3, and M 4 (Zimanyi-Moszkowski model). Full and dashed lines correspond to set I and set II of parameters, respectively. The quark mass at zero baryon density has been taken as $m_q = 10$ MeV.

Figure 2: $B^{1/4}$ in [MeV] as a function of the relative baryon density ρ/ρ_0. The conventions and parameters used are the same as in Fig.1.

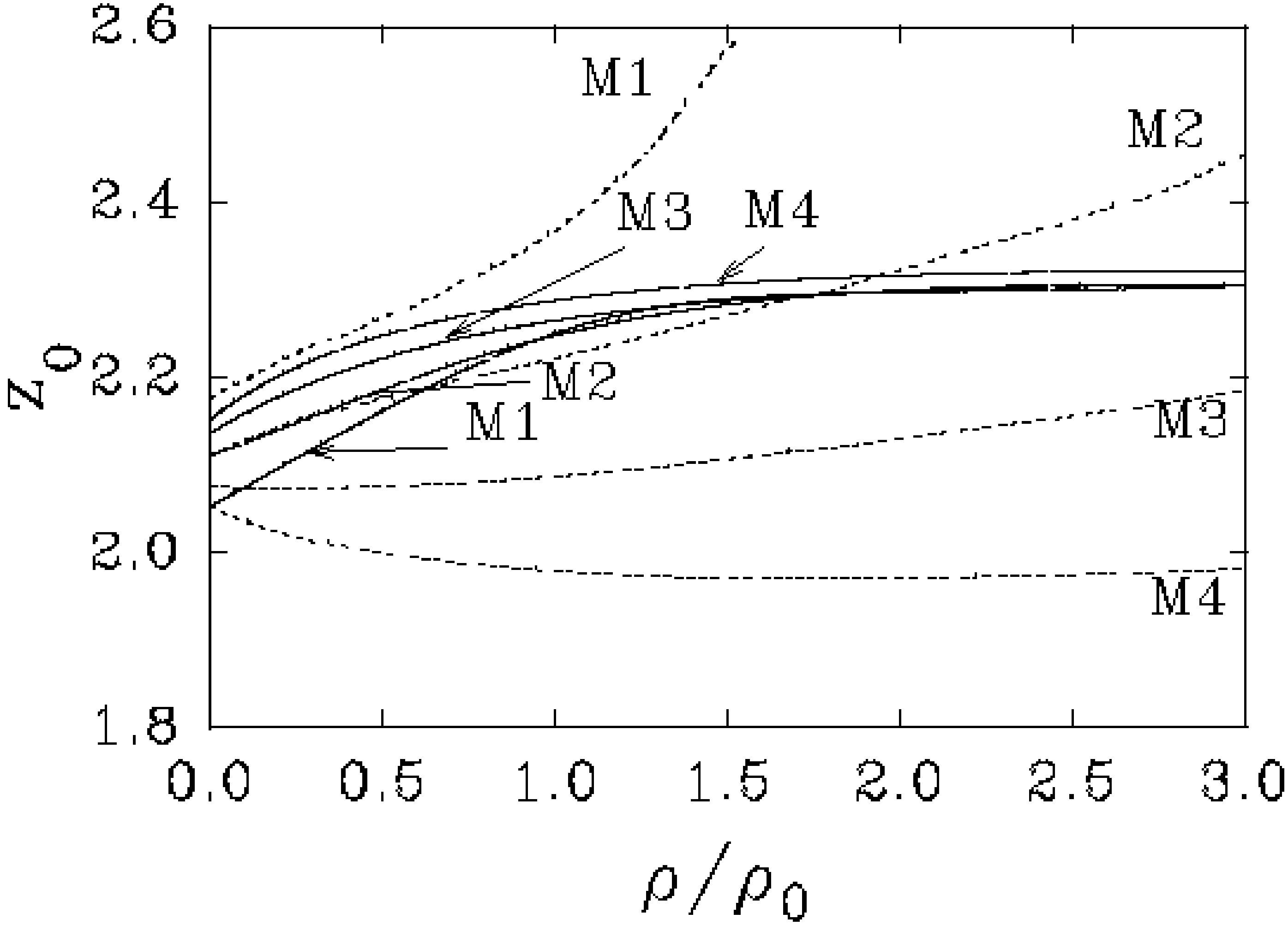
Figure 3: z_0 as a function of the relative baryon density ρ/ρ_0. The conventions and parameters used are the same as in Fig.1.



R [fm]
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ρ/ρ_0
M4
M3
M1
M2
M3
M4
M2
M1



250
200
150
100
50
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$B^{1/4}$ [MeV]
ρ/ρ_0
M1
M1
M2
M2
M3
M4


M1
M2
M3
M4
M2
M1
M3
M4
2.6
2.4
2.2
2.0
1.8
z_0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ρ/ρ_0